Bandpass Sigma-Delta Analog-to-Digital Conversion

Stephen Jantzi, Richard Schreier, and Martin Snelgrove

Abstract-The traditional low-pass ΣΔ analog-to-digital converter is extended to the bandpass case. For input signals with small relative bandwidths, bandpass ΣΔ converters offer high signal-to-noise ratios at significantly lower sampling rates tban are required for low-pass ΣΔ converters. A sixth-order single-ended switched-capacitor circuit, clocked at 3 MHz, is designed to convert bandpass signals centered at 455 kHz with 20-kHz bandwidth. Time-domain circuit simulations find that this modulator realizes a 94-dB signal-to-noise ratio for a half-scale input, giving roughly 16-bit performance.

I. INTRODUCTION

Sigma-delta modulation has recently become the method of choice for high-resolution A/D conversion. The benefits of oversampled noise-shaping converters include inherent linearity, reduced anti-aliasing filter complexity, high tolerance to circuit imperfection, and a system architecture that lends itself to switched-capacitor implementation [l]-131. The bandpass variant of ΣΔ retains these advantages and offers a promising technique for use in the rapidly developing area of digital ratio.

Traditional ΣΔ converters place noise transfer-function zeros near $\omega_o = 0$ in order to null quantization noise in a narrow band around dc. This noise-shaping concept was extended in [4] to the bandpass case, wherein the noise transfer-function zeros are placed at a nonzero frequency, ω_o. Quantization noise is nulled in a narrow band around ω_o, such that the output bit-stream accurately represents the input signal in this narrow band.

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TABLE I
DESIGN PARAMETERS FOR OUR SIXTH-ORDER BP ΣΔ MODULATOR

Parameter	Symbol	Value	Normalized Value
center frequency	f_o	455 kHz	$455\pi/1500$
bandwidth	f_b	20 kHz	$\pi/75$
sampling frequency	f_s	3 MHz	2π
oversampling ratio	R	$75\,(f_s/2f_b)$	

For narrow-band signals away from dc, the band-reject noise-shaping of a bandpass ΣΔ converter results in high signal-to-noise ratios (SNR'S) at significantly lower sampling rates than required for low-pass ΣΔ converters. The oversampling ratio, R, is defined as one-half the sampling rate divided by the width of the band of interest. Imagine the conversion of a signal centered at 1 MHz with l0-kHz bandwidth. With a l0-MHz sampling rate, a traditional converter would provide five times oversampling; a bandpass ΣΔ converter would achieve 500 times oversampling.

Bandpass ΣΔ A/D converters are well suited for use in the front end of radio receivers, allowing direct conversion to digital at either intermediate- or radio-frequency. An early conversion to digital results in a more robust system with improved IF-strip testability. Additionally, it provides opportunities for dealing with the multitude of standards present in commercial broadcasting and telecommunications. Thispaper examinedbandpass ΣΔ converters, specifically the feasibility of their implementation using standard switched-capacitor circuit techniques.

II. TRANSFER-FUNCTION DESIGN

2, I. *Choice of Band Location*

In this discrete-time system, a choice can be made in the location of the band-of-interest, Trade-offs are possible among sampling rate, oversampling ratio, and anti-aliasing filter requirements. Placing the band near $\omega=\pi$ reduces the sampling rate, but increases the demands on the anti-aliasing filter and reduces the oversampling ratio. Moving the band closer t$\omega=0$ can increase the sampling rate to an unreasonable level.

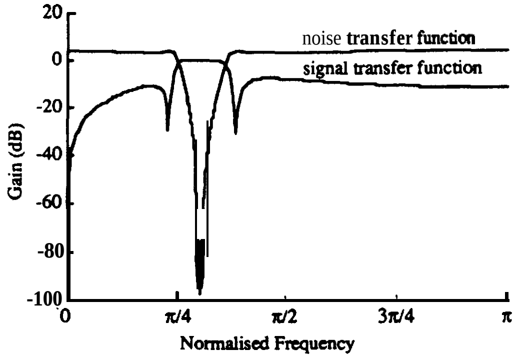
We wish to convert a signal centered at 455 kHz with 20-kHz bandwidth. A compromise is made in our choice of band location, and with a sampling rate of 3 MHz we achieve 75 times oversampling. A summary of our design values is given in Table I.

Other issues to consider in the choice of band location concern the digital post-filtering and decimation. A method for simple decimation and demodulation has been described in [5] that is well suited to band placement at simple fractions of π. For the band location chosen herein, brute-force single-stage ROM-based FIR decimation can be used [6].

2.2. *The Noise Transfer Function*

In the analysis of ΣΔ modulators, the quantizer is often replaced by an additive noise source to yield a linear model. The input signal, u, and the noise of the quantizer, n, have different z-domain transfer functions, $H_u(z)$ and $H_n(z)$, respectively, to the output, y.

The noise transfer function (NTF), $H_n(z)$, is chosen to provide maximum in-band attenuation, subject to several constraints. Firstly, $H_n(\infty)$ must equal one to guarantee realizability. Secondly, the maximum gain of $H_n(z)$ must not exceed 2,



	NTF	STF
zeros	0.59248 ± 0.80537j 0.57921 ± 0.81464j 0.56596 ± 0.82422j	0.75011 ± 0.66131j 0.35347 ± 0.93544j 1
poles	0.43068 ± 0.54366j 0.68315 ± 0.66507j 0.37609 ± 0.86022j	

Fig. 1. Noise and signal transfer functions for a sixth-orderbandpass modulator centered at $455\pi/1500$.

following a rule-of-thumb' for stability proposed by Lee [7]. A margin of safety is maintained by limiting the maximum gain of $H_n(z)$ to 1.6.

In-house filter design software facilitates the design cH_n [8]. A least-pth optimizer adjusts the poles and zeros of H_n such that its amplitude response closely matches user-defined measures of ideality. The design requirements can be entered as three separate ideal responses:

i) in-band: infinite attenuation;
ii) out-of-band: $|H_n| < 1.6$ (4 dB);
iii) infinity: $H_n = 1$.

Fig. 1 shows the optimized NTF of the sixth-order modulator specified by Table I. It is band-reject, has an approximately equiripple stopband with 76.5-dB attenuation across the band of interest, and has 4 dB out-of-band gain.

2.3. *The Signal Transfer Function*

The cascade-of-resonators structure chosen for implementation has been described previously in[l0], and is shown in Fig. 2. The signal transfer function (STF) shares poles with the noise transfer function, but has one less zero. As such, its shape can be quite limited for low-order modulators. Modulators of order six and above allow the design of satisfactory STF's: constant gain and linear phase in-band, high attenuation and no gain peaks out-of-band. Out-of-band attenuation is helpful as it reduces internal signal levels due to large out-of-band inputs, and may improve stability. Again, the least-pth optimizer is useful for optimal placement of zeros to meet the above criteria.

Fig. 1 shows the optimized STF. It is bandpass with a wide, flat (within 0.005 dB) passband extending well beyond the band of interest, and approximately 10 dB of stopband attenuation.

'This rule-of-thumb is both unnecessary and insufficient [9], but is easy to use and often correct, especially when a good safety margin is used.

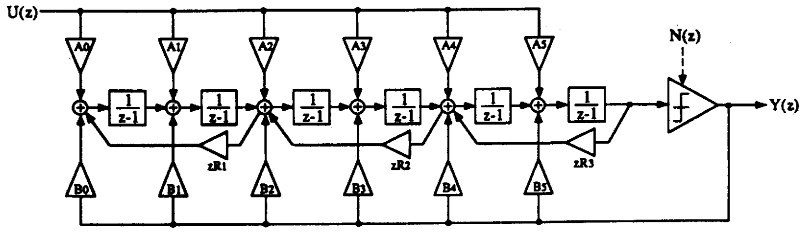


Fig. 2. The 6th-order cascade-of-resonators structure.

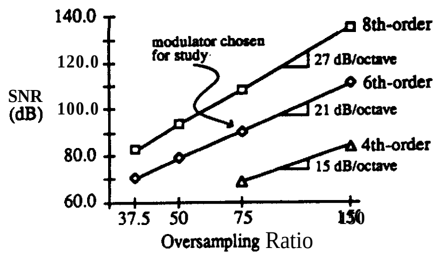


Fig. 3. Simulated SNR ratio performance for 4th-, 6th-, and 8th-order BP ΣΔ modulators. The input was a tone, 10 dB below full scale.

III. Modulator Performance

A detailed derivation relating SNR, oversampling ratio, and modulator order can be found in [4]. For each octave increase in the oversampling ratio we realize a $3n + 3$-dB increase in SNR, where n is the modulator order. The $3n$ term, as opposed to $6n$ for low-pass ΣΔ, is a result of the fact that an nth-order bandpass ΣΔ modulator has in-band zeros of multiplicity n /2.

Optimized NTF's were designed for several fourth-, sixth-, and eighth-order bandpass modulators with center frequency 455 kHz and bandwidths 10, 20, 30, and 40 kHz (oversampling ratios of 150, 75, 50, and 37.5, respectively). The results of simulations for a – 10-dB input signal (relative to full-scale) are plotted in Fig. 3. This figure clearly shows the expected relationship between SNR and oversampling ratio.

IV. Capacitor Matching and Sizing

4. I. *Sensitivity to Capacitor Mismatch*

Capacitor ratios directly affect the coefficients of the structure shown in Fig. 2. The coefficients in turn determine the signal and noise transfer functions. The R coefficients set the zero locations of the NTF, the A and R coefficients set the zero locations of the STF, and the B and R coefficients set the pole locations of both the STF and NTF. Thus capacitor mismatch alters the NTF, which reduces in-band attenuation of noise and lowers the effective SNR.

An analytical calculation of in-band noise power is possible using the linear model and assuming a noise spectral density of 1/3. For small deviations in circuit capacitor ratios, the modified noise transfer function can be found and the in-band noise power calculated, It has been found that the two most important coefficients are the R coefficients which determine the location of the band-edge zeros, while the R coefficient which determines the band-center zero is slightly less critical, and the remaining coefficients are relatively unimportant. This is intuitively satisfying because it is the location and quality of the NTF zeros that most affect the amount of in-band noise power.

To simulate effects likely to occur during manufacturing, all capacitor ratios were repeatedly varied by up to 0.5%, and the noise power calculated. With this tolerance, which is readily obtainable in current switched-capacitor technology, 95% of the modulators had less than 2-dB degradation in SNR. The effect of capacitor mismatch on modulator stability was studied by examining the maximum NTF gain, $\|H_n\|_\infty$. The optimum modulator had a maximum gain of 1.56, and 95% of the modulators tested retained maximum gains below 1.64, only a 5% increase over the design value. Both of these tests examined a set of 50 000 perturbed modulators.

4.2. *Scaling for Maximum Dynamic Range*

Capacitor ratios were chosen to correctly set the A, B, and R coefficients, and then adjusted to scale the circuit for maximum dynamic range. As the BPΣΔ concept is new, and the effects of clipping are not yet well understood, a conservative l_∞-norm was used: scaling was based on the worst-case voltage swings at the integrator outputs.

4.3. *Absolute Sizes*

Minimum capacitor sizes are determined by noise constraints. The kT/C noise present on each switched capacitor is assumed to have a white spectrum, and is spread from 0 to 7. As we are concerned strictly with in-band noise, we gain an oversampling-factor reduction in this kT/C noise power. This allows the minimum allowable capacitor size to be reduced by the same factor, and proves to be an added benefit of oversampling. Additionally, certain capacitors see noise-shaped transfer functions to the output, again reducing output noise power, and hence, allowable capacitor size. As a result of these considerations, the first resonator stage, which is responsible for the majority of the kT/C noise at the output, needs the largest capacitors and these can be as low as 1 pF in a fully differential design.

Maximum capacitor sizes are determined by bandwidth and slew-rate constraints. Given specific op-amp performance, capacitors must be sized to allow sufficient amplifier settling.

V. Operational Amplifiers

5.1. *Finite dc Gain*

Operational amplifiers with high dc gain are necessary to maintain the desired transfer functions. Each resonator in the structure includes two op-amps and is responsible for making a notch at a specific frequency. Finite op-amp gain in a resonator shifts both the radius and the angle of the transfer-function zero, thereby reducing in-band attenuation and increasing in-band noise at the modulator output. The transfer-function poles also shift, which can further reduce in-band attenuation.

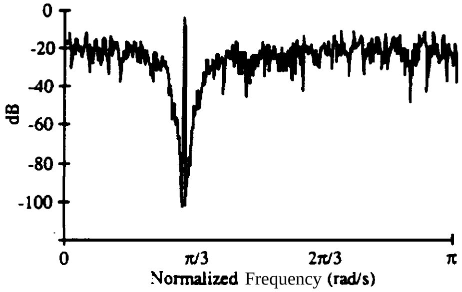


Fig. 4. Output spectrum for a BP ΣΔ modulator with half-scale tone input (Hann-windowed 1024-bin FFT).

The effect of finite dc gain, A, on a notch angle is similar to the effect of a capacitor mismatch. At our chosen band location, a 60-dB dc gain, which reduces $(1-1/A)$ by 0.1%,[2] increases notch frequency by 0.054%. A 0.1% increase in the appropriate capacitor ratio likewise increases notch frequency by 0.054%. Design centering can be effected by predistorting the capacitor values to cancel the effects of finite dc gain on notch frequency.

A 60-dB dc gain decreases notch radius by the full O.l%, a shift that predistortion of capacitor ratios cannot compensate for.

The total effect of 60-dB op-amp gain in one resonator is an increase in noise power of approximately 0.2 dB. Finite 60-dB gain in all six op-amps, without predistortion of capacitor values, reduces the SNR ratio by less than 0.4 dB.

5.2. *Speed*

Satisfactory op-amp bandwidth and slew-rate performance are necessary to ensure that sufficient settling occurs. These values depend both on op-amp size and bias current, and, as mentioned earlier, on capacitor size. As is the case with low-pasΣΔ converters, it is possible that integrator outputs will change by their maximum swing in each clock cycle. Thus, settling constraints are based on sampling rate, not on input signal frequency, and consequentlybandpass ΣΔ converters have settling requirements no more stringent than those of low-pass ΣΔ converters.

VI. Simulation Results

The linear model predicts an SNR of 91.0 dB for a half-scale input. The 76.5-dB in-band attenuation of noise shown in Fig. 1, combined with 75 times oversampling, results in this SNR. For the same input level, a 93.3-dB SNR results for time-domain simulation of an ideal mathematical model of the modulator.

A time-domain circuit simulation was performed on a single-ended switched-capacitor version of the modulator [ll]. A 1024-bin Hann-weighted FFT of the output bit-stream is plotted in Fig. 4, which clearly shows the band-reject noise-shaping. Analysis on the bit-stream found an SNR of 93.8 dB for a half-scale tone input.

[2] A switched-capacitor integrator has a pole at z = 1 -(l/A), which moves to $z = 1$ for infinite op-amp gain, A.

VII. Conclusions

The design of bandpass ΣΔ analog-to-digital converters has been discussed. Simulations have shown the expected $3n+3$-dB increase in the SNR for each octave increase in oversampling ratio. Sensitivity analysis has shown that random 0.5% capacitor mismatch causes only a 2-dB reduction in the SNR for the cascade-of-resonators structure we have chosen. A finite dc gain of 60 dB in all op-amps is seen to be adequate, reducing the SNR by less than 0.4 dB. Op-amp speed has been discussed, and it was seen that similar op-amp settling performance is required in bandpass and low-pass ΣΔ modulators.

It has been shown that a structure exists for a switched-capacitor implementation of a bandpass ΣΔ modulator that maintains a high conversion accuracy in the face of circuit nonidealities.

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