Bandpass Sigma-Delta Modulation

Abstract

A bandpass version of sigma-delta modulation is presented, with simulation results for second and fourth-order converters. For the fourth order converter operating at 8MHz, simulations demonstrate a resolution of 16 bits over an 8kHz band centred at 1MHz. Applications may include analog to digital conversion for AM radio, and digital to analog conversion for narrow-band RF systems.

Introduction: Sigma-delta modulation[1,2,3] is a technique for doing both analog-to-digital and digital-to-analog conversion that uses very simple analog components and digital signal processing to achieve high accuracy and an immunity to component errors. Thus far, research has focused on systems wherein the sampling rate is much greater than the highest frequency component of the input. This letter extends the range of application of sigma-delta modulation by demonstrating that the sampling rate need only be much greater than the bandwidth of the input. The resulting modulators are dubbed bandpass sigma-delta modulators.

A basic premise of sigma-delta modulation is that the sampling rate is much greater than the highest frequency of interest present in the input. This is necessary because ordinary (lowpass) sigma-delta modulators zero quantization noise only near DC. If one were to null quantization noise at some other frequency, say ω_0, then one would get good accuracy in a band around ω_0. Figure 1 illustrates the pole and zero placement of the error transfer functions for lowpass and bandpass sigma-delta modulators, and highlights the respective passbands.

A narrow bandpass post-filter, centred at ω_0, is required to attenuate the out-of-band quantization noise, in correspondence with the narrow lowpass post-filter needed by ordinary sigma-delta modulation. The resultant system should possess all the favourable characteristics of regular sigma-delta modulation, namely inherent linearity and tolerance to component variations.

Design: We begin the design of a modulator with the selection of $\boldsymbol{H}$, the error transfer function. By reversing the conventional analysis procedure, it is possible to derive a modulator that realizes $\boldsymbol{H}$ provided $\boldsymbol{H} - 1$ is strictly causal, i.e. $H(\infty) = 1$. This letter presents results for two choices of $\boldsymbol{H}$:

$$H_1 = (1 - \sqrt{2}z^{-1} + z^{-2})$$

$$H_2 \;=\; \frac{(z^2-\sqrt{2}z^1+1)^2}{z^4-2.1757z^3+2.3077z^2-\ 1.30549\ +.3846}$$

The first is second-order and has single zeros at $\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$, i.e. $\frac{1}{8}f_s$, and two poles at $z = 0$. This modulator mimics the behaviour of a regular first-order sigma-delta modulator in that the zero of H in the centre of the band is a first-order zero. The second modulator is fourth order, with zeros of multiplicity two, and so corresponds to a second-order lowpass sigma-delta modulator. It is necessary to move the poles away from the origin for a fourth-order modulator to be stable. The poles were chosen such that $\max_{\omega\in[0,\pi]}|H(e^{j\omega})| = 1.625$, following the technique presented by Lee.[4] In brief, Lee claims that the condition $\max_{\omega\in[0,\pi]}|H(e^{j\omega})| < 2$ ensures that the resulting modulator is stable, and suggests the use of error transfer functions with a monotonic magnitude response. H_2 satisfies these requirements. It is a straightforward task to design higher order converters using Lee's technique.

Theory: Near a zero of order n, the magnitude of the error transfer function is

$$|H(e^{j\omega})| = k(\omega-\omega_0)^n, \qquad \text{where} \quad k = \frac{1}{n!}\frac{d^n|H(e^{j\omega})|}{d\omega^n}\bigg|_{\omega=\omega_0}. \tag{1}$$

The noise power on one side of this zero is given by

$$N^2 = \int_{\omega_0}^{\omega_0+\omega_B} |H(e^{j\omega})|^2 \, \frac{\bar{e^2}}{\pi} \, d\omega \tag{2}$$

$$= \frac{k^2 \omega_B^{2n+1}}{3\pi(2n+1)}. \tag{3}$$

We have assumed that the above approximation for $|H|$ holds, and that the quantization error is white and uniformly distributed over [-1, **+l], so** that $\bar{e^2} = \frac{1}{3}$. If we identify the oversampling ratio as $R = \frac{\pi}{\omega_B}$, and assume an input signal power of $\frac{a^2}{2}$, corresponding to a sine wave of amplitude *a,* then the signal-to-noise ratio of an n^{th} order lowpass converter is:

$$\text{SNR} = 10 \log \frac{3a^2(2n+1)R^{2n+1}}{2k^2\pi^{2n}} \text{ dB}. \tag{4}$$

For a bandpass converter, we need to integrate the noise from $\omega_0 - \frac{\omega_B}{2}$ to $\omega_0 + \frac{\omega_B}{2}$, and the order of each zero is half the order of the converter. Thus the signal-to-noise ratio of an n^{th} order bandpass converter is:

$$\text{SNR} = 10 \log \frac{3a^2(n+1)(2R)^{n+1}}{4k^2\pi^n} \text{ dB} \tag{5}$$

Simulations: The operating band was taken to be $\frac{1}{2}f_s$ **X** $(\frac{1}{4} \pm \frac{1}{1024})$, corresponding to an oversampling ratio of 512. Figure 2 plots the signal-to-noise ratio for both modulators against input amplitude. The simulations were done by injecting single tones into the converter for 2^{18} samples and then doing a discrete Fourier transform with a Hanning window to find the ratio of the tone power to the in-band noise power.

According to Equation 5 (substituting n = 2, R = 512, $k = \sqrt{2}$), *we* expect a SNR of 60.8dB for the second order modulator when the input is -20dB. The simulations yield nearly 60dB. For the fourth order modulator, (n=4,R=512,k=l3) we expect 94.1dB and the simulations show a SNR of 99dB.

Applications: It is believed that a bandpass sigma-delta modulator can be constructed to perform analog-to-digital conversion for narrowband signals such as those found in AM radio. For the fourth order modulator presented here, an 8MHz sampling rate is enough to achieve a resolution of better than 16 bits at lMHz, over an 8KHz bandwidth. It is assumed that the digital filter can be made sufficiently narrow and noise-free to achieve the performance predicted by the simulations. Note .that we do not require a narrowband front end: the digital filtering to eliminate sigma-delta noise also handles out-of-band signals.

Another possible application is in digital to analog conversion for RF systems. Here the modulator is digital and the bandpass filter is analog, a tuned circuit for example. The bi-level coding provided by sigma-delta modulation allows the driving circuitry to be operated in a switching mode, which is known to be highly energy efficient. The noise-shaping property may result in an improvement over pulse-width modulation.

Conclusions: Simulation results for an extension of sigma-delta modulation, bandpass sigma-delta modulation, were presented. It was suggested that bandpass sigma-delta modulation could be applied to the analog-to-digital and digital-to-analog conversion of high-frequency narrowband signals.

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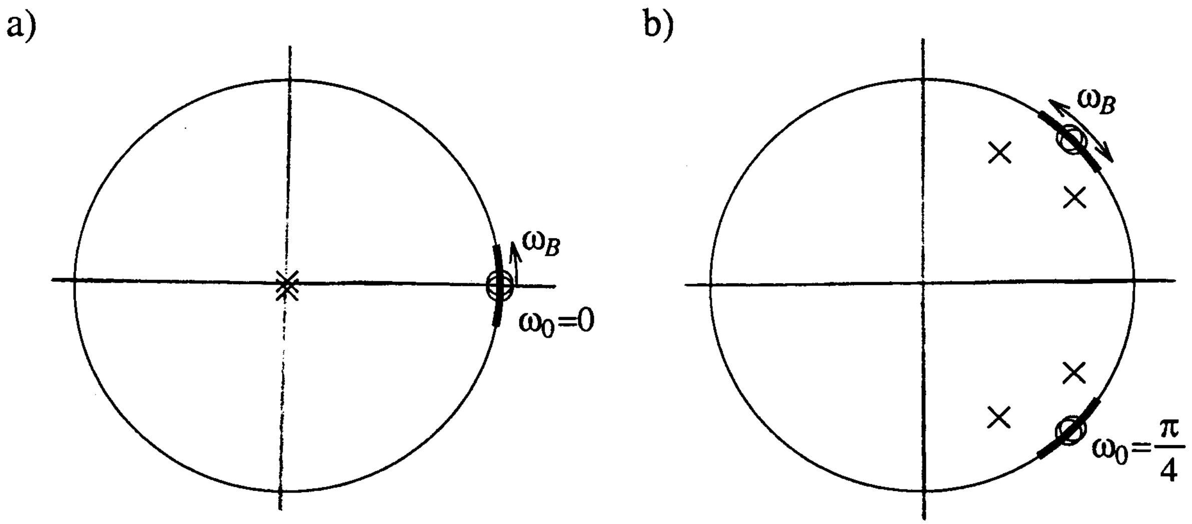

Figure **1:** Comparison of the pole and zero placements of the error transfer function for a) an ordinary second-order lowpass sigma-delta modulator b) a fourth-order bandpass sigma-delta modulator. The passbands are highlighted.

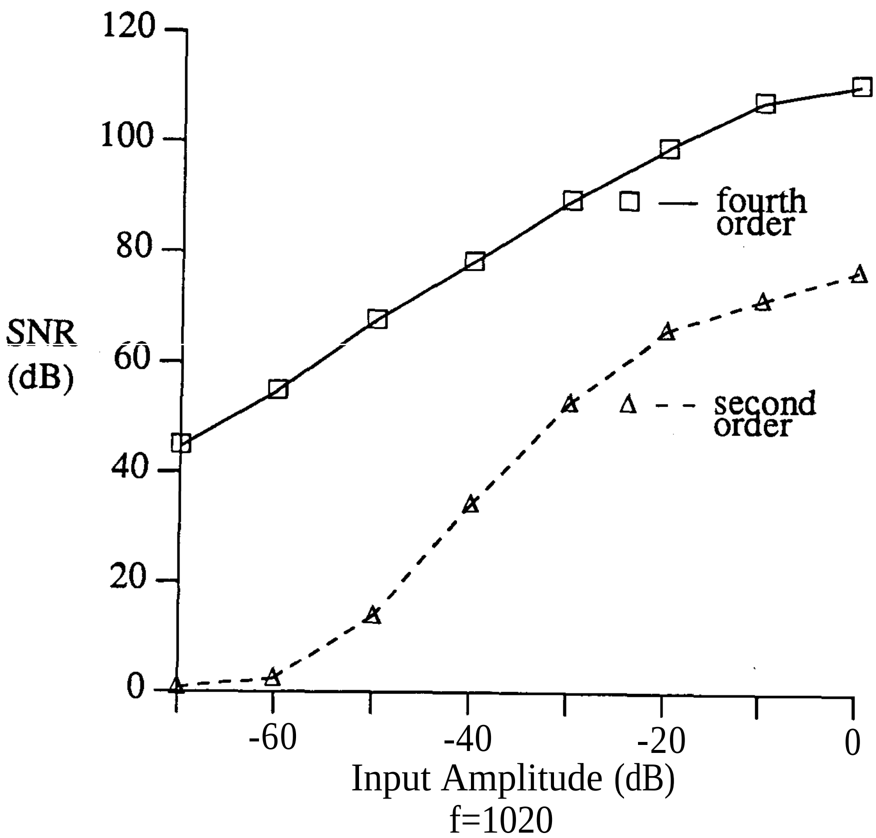

Figure 2: SNR versus input amplitude for second- and fourth-order modulators.