Adaptive Linearization of A Loudspeaker

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ABSTRACT

This paper presents a new and promising application for adaptive nonlinear filters - adaptive linearization of a loudspeaker. In a loudspeaker, the nonlinearity in the suspension system produces a significant distortion at low frequencies and the inhomogeneity in the flux density causes a nonlinear distortion at large output signals. These distortions should be reduced so that high fidelity sound can be reproduced. The conventional feedback technique has difficulties combating the nonlinear distortions due to an air path delay in the feedback signal. An adaptive approach may lead to a good solution. An adaptive pre-distortion linearization scheme, together with other schemes, was recently presented in [1] for a weakly nonlinear system. This paper employs the pre-distortion scheme for linearizing a loudspeaker. A model of a direct radiator loudspeaker has been developed and studied which takes into account the two principal sources of nonlinear distortions. Based on this model, simulations of the proposed method have been performed. The results have shown that nonlinear distortions of a loudspeaker can be reduced significantly.

I. INTRODUCTION

The principal causes of nonlinearities in a loudspeaker include nonlinear suspension and non-uniform flux density [11-13]. The suspension nonlinearity affects distortion mainly at low frequencies. At frequencies of about $300Hz$ or above, the total harmonic distortion of a loudspeaker is usually fairly low (of the order of 1%) and not appreciably affected by the suspension nonlinearity. As the frequency decreases, however, distortion rises rapidly in loudspeakers having a suspension nonlinearity. For instance, a 10 inch dynamic loudspeaker with a nonlinear suspension has been measured, producing 10% total harmonic distortion with an input of 2 watts at $60Hz$ [13]. The distortion caused by non-uniform flux density is small, usually less than 1%, as long as the amplitude of movement is small. However, the distortion is severe if the output signals are large. These distortions can be reduced by careful design using some conventional techniques. This paper proposes an adaptive pre-distortion approach, which can be used in addition to the conventional design approaches and may result in a substantial reduction in nonlinear distortions. This approach is based on a recently presented adaptive linearization scheme for weakly nonlinear systems [1]. In this paper, after discussing the adaptive linearization scheme, a model of a loudspeaker with a suspension nonlinearity and non-uniform flux density is derived and studied, then simulation results are presented. It should be stressed that although the paper studies the two principal causes of nonlinearity in a loudspeaker, the proposed adaptive linearization method is also expected to linearize a loudspeaker with other nonlinearities.

On the other hand, linearization of a loudspeaker is a new application of adaptive nonlinear filters. Adaptive nonlinear filters have been studied for some time, and some structures and algorithms have been developed, e.g. [2-10]. However, the applications are still quite limited. New applications will certainly be stimulating.

II. THE ADAPTIVE PRE-DISTORTION SCHEME

Three linearizations schemes were presented in [1]: linearization by cancellation at the output, linearization with a post-processor, and linearization with a pre-processor. The scheme of linearization by cancellation at the output and the scheme with a post-processor are not suitable for a loudspeaker application because these schemes require processing of sound signals after sound waves leave the loudspeaker. Processing of sound is difficult. The scheme with a pre-processor handles signals in electrical form. Hence, it can be easily realized using the DSP technique.

In the following discussion, inverse modeling of the linear behavior of a nonlinear system will be used. Let L_l indicate the linear operator of a loudspeaker and L^{-1} indicate the linear operator obtained by an adaptive linear filter which performs inverse modeling of this loudspeaker. Then, we can have L^{-1}, satisfying

$$L^{-1}L_l = z^{-\delta} \tag{1}$$

where $z^{-\delta}$ indicates a delay of δ samples and δ usually must be nonzero so that the adaptive filter can converge.

A nonlinear processor will be designed to pre-distort signals, as shown in Fig.1. A nonlinear processor with the following nonlinear mapping

$$y_i(k) = u(k-\delta) - L^{-1}(N(u)) \tag{2}$$

can perform the task, where u is the input signal, k is the time, and N is an estimate of the nonlinear operator N_l of the loudspeaker. This can be verified easily. According to Volterra theory, the output y_l of a loudspeaker system, which has weak nonlinearities, can be expressed as a sum of a linear signal and a nonlinear signal [1]

$$y_l(k) = L_l(u(k)) + N_l(u(k)) \tag{3}$$

where L_l and N_l are linear and nonlinear operators of the loudspeaker system, respectively. Hence, the output of the loudspeaker is

$$y_l(k) = L_l(y_i(k)) + N_l(y_i(k)) \tag{4}$$

Substituting Equation (2) into the above equation and employing Equation (1) results in

$$y_l(k) = L_l(u(k-\delta)) - N(u(k-\delta)) + N_l(u(k-\delta) - L^{-1}(N(u(k)))) \tag{5}$$

Assuming that the nonlinearity is weak, namely

$$|L_l(u(k))| \gg |N_l(N(u(k)))| \tag{6}$$

or

$$|u(k-\delta)| \gg |L^{-1}(N_l(N(u(k))))| \tag{7}$$

and assuming the operator N_l is smooth, we have

$$y(k) = L_l(u(k-\delta)) \tag{8}$$

Hence, the loudspeaker output is the linearized output, namely, $y_l = y_{linearized}$.

The ratio of the linear signal to the residual nonlinear distortion can be estimated. Because of the assumption of weak nonlinearity in

Equation (7), the loudspeaker's nonlinear part can be approximated by

$$N_l(u(k-\delta) - L^{-1}(N(u(k)))) = N_l(u(k-\delta)) - N_l'(u(k-\delta))L^{-1}(N(u(k))) \quad (9)$$

where the nonlinear operator N_l' is the derivative of the nonlinear operator N_l. Considering the above equation and Equation (5), we can see the ratio of the linear signal to the residual nonlinear distortion is

$$\gamma = \frac{||L_l(u(k-\delta))||_2}{||N_l'(u(k-\delta))L^{-1}(N(u(k)))||_2} \quad (10)$$

To gain some insight, let us suppose the original nonlinear signal is weaker by α, where α is greater than one. Then, after linearization the ratio becomes

$$\gamma = \alpha^2 \frac{||L_l(u(k-\delta))||_2}{||N_l'(u(k-\delta))L^{-1}(N(u(k)))||_2} \quad (11)$$

This shows that if the original distortion is smaller by α, the ratio is increased by α^2 after linearization.

A simple example can help further illustrate the ideas. Suppose that the loudspeaker system is described by a Taylor series, the memoryless case of a Volterra series,

$$y_l = 2y_i(k) + 0.06y_i^2(k) \quad (12)$$

and assume that the norms of the signals y_i and u are not greater than unity so that the magnitudes of the nonlinearities can be roughly determined by the magnitudes of the nonlinear coefficients. The pre-processor can be designed using Equation (2),

$$y_i(k) = u(k) - L^{-1}(N(u))$$

where the delay δ is zero due to memorylessness. Then,

$$y_{linearized}(k) = 2(u(k) - 2^{-1}(0.06u^2(k))) + 0.06(u(k) - 2^{-1}(0.06u^2(k)))^2$$

$$= 2u(k) - 0.0036u^3(k) + 0.000054u^4(k)$$

$$\doteq 2u(k) \quad (13)$$

This example demonstrates that the pre-processor is able to reduce the nonlinear distortion. Assuming the norm of the input signal u is unity, the original ratio of linear signal to distortion is

$$\gamma = \frac{2}{0.06}$$

After linearization, this ratio becomes

$$\gamma = \frac{2}{0.0036}$$

The ratio in dB is almost doubled by the linearization technique.

The adaptive implementation of the linearization scheme is shown in Fig.2, where the adaptive filters provide the necessary estimates. The linear FIR filter of the pre-processor is copied from the adaptive linear FIR filter and the nonlinear operator N of the pre-processor is copied from the nonlinear part of the adaptive nonlinear FIR filter. At the beginning of adaptation, the adaptive filters do not give good estimates of the operators so that the pre-processor may not reduce the nonlinearity in the loudspeaker, but may worsen it. Hence, it is better to copy after the adaptive filters have run for some time and have good estimates.

In this paper, the adaptive linear FIR filter for inverse modeling is a transversal FIR filter:

$$y(k) = \sum_{i=0}^{n} h(i)u(k-i) \quad (14)$$

where n is the filter order and $h(i)$ are the filter coefficients. The coefficients are updated using the LMS algorithm:

$$h^{k+1}(i) = h^k(i) + 2\mu e(k)u(k-i) \quad (15)$$

where μ is the step size and e is the error, defined as the difference between the reference signal and the filter output.

The adaptive nonlinear FIR filter, whose purely nonlinear part provides an estimate of the nonlinear operator N_l, is a Volterra filter:

$$y(k) = \sum_{i_1=0}^{n_1} h_1(i_1)u(k-i_1) + \sum_{i_1=0}^{n_2}\sum_{i_2=i_1}^{n_2} h_2(i_1,i_2)u(k-i_1)u(k-i_2) + \cdots$$

$$+ \sum_{i_1=0}^{n_m}\sum_{i_2=i_1}^{n_m}\cdots\sum_{i_m=i_{m-1}}^{n_m} h_m(i_1,i_2,\cdots,i_m)u(k-i_1)u(k-i_2)\cdots u(k-i_m) \quad (16)$$

where m is the total number of terms in the filter. The sum with h_1 is linear in terms of the input signal u. The sum with h_i will be referred to as ith power term. Particularly, terms with h_2 and h_3 will be referred to as quadratic term and cubic term, respectively. n_1 is the order of the linear term, similarly, $n_2, n_3, ...,$ and n_m are the orders of the corresponding terms.

Updating the coefficients of the Volterra filter is performed as follows [7,8]:

$$h_j^{k+1}(i_1,i_2,\cdots,i_j) = h_j^k(i_1,i_2,\cdots,i_j) +$$

$$2\mu_j e(k)u(k-i_1)u(k-i_2)\cdots u(k-i_j) \quad (17)$$

where $j = 1, 2, , ..., m$ and μ_j is the step size for the jth power term.

III. MODELING OF A LOUDSPEAKER SYSTEM

The basic direct radiator loudspeaker is chosen for study due to its simplicity and popularity. The results developed in this paper can be generalized to horn loudspeakers and variants of the basic direct radiator loudspeaker.

Equivalent Circuit of A Loudspeaker

A loudspeaker is composed of an electrical part and a mechanical part as shown in Fig.3. The electrical part is simply the voice coil. The mechanical part consists of the cone, the suspension, and the air load. The two parts interact through the magnetic field. The mechanical part can also be described by an equivalent electrical circuit, which will be called the mechanical circuit.

The electrical circuit and mechanical circuit of a loudspeaker are shown in Fig.4 [12,14]. In the voice coil electrical circuit, e indicates the internal voltage of the generator, r represents the total electrical resistance of the generator and the voice coil, L is the inductance of the voice coil, i is the amplitude of the current in the voice coil, E is the voltage produced in the electrical circuit by the mechanical circuit and $E = Bl dx/dt$, where B is the magnetic flux density in the air gap, l is the length of the voice coil conductor, and x is the cone displacement. In the mechanical circuit, m represents the total mass of the coil, the cone and the air load, r_M indicates the total mechanical resistance due to dissipation in the air load and the suspension system, C_M is the compliance of the suspension, and f_M is the force generated in the voice coil and is equal to Bli.

In terms of analogies, the dimensions in the electrical circuit corresponding to length, mass, force and time in the mechanical system are charge, self-inductance, generator voltage, and time. Thus, we can write the differential equation for the mechanical circuit:

$$m\frac{d^2x}{dt^2} + r_M\frac{dx}{dt} + \frac{x}{C_M} = Bli \quad (18)$$

Referring to the electrical circuit shown in Fig.4, the following equation can be written:

$$e = ir + L\frac{di}{dt} + Bl\frac{dx}{dt} \quad (19)$$

Distortions in a Loudspeaker

Generally, the mechanomotive force in the voice coil is a non-linear, instead of linear, function of displacement, so that the compliance of the suspension system is a function of the displacement instead of a constant. The force deflection characteristic of the loudspeaker cone suspension system can be usually approximated by a polynomial

$$f_M = \alpha x + \beta x^2 + \gamma x^3 \qquad (20)$$

where f_M indicates the applied force which causes the displacement x. Then, the compliance of the suspension system can be obtained

$$C_M = \frac{x}{f_M} = \frac{1}{\alpha + \beta x + \gamma x^2} \qquad (21)$$

thus, substituting above equation into (18), we have

$$m\frac{d^2 x}{dt^2} + r_M\frac{dx}{dt} + \alpha x + \beta x^2 + \gamma x^3 = Bli \qquad (22)$$

The above equation shows that at high frequencies the derivatives are large so that the effect of the nonlinearity is weak and the equation is more linear, while at low frequencies the derivatives are small so that the effect of the nonlinearity is strong and the equation is more nonlinear. This is why the distortion is more severe at low frequencies for a loudspeaker with a suspension nonlinearity.

Another source of harmonic distortion is non-uniform flux density up to the maximum amplitude of operation. The flux density B is not a constant, instead it is a function of the displacement x, which may also be approximated by a polynomial

$$B(x) = B_0 + B_1 x + B_2 x^2 \qquad (23)$$

This nonlinearity affects both the electrical circuit and the mechanical circuit, as suggested by Equations (18) and (19).

Let $x_1 = i$, $x_2 = x$, and $x_3 = dx_2/dt$, then, we have the following state-space equation

$$\frac{dx_1}{dt} = \frac{1}{L}(-rx_1 - B_0 lx_3 + e - B_1 lx_2 x_3 - B_2 lx_2^2 x_3)$$

$$\frac{dx_2}{dt} = x_3$$

$$\frac{dx_3}{dt} = \frac{1}{m}(B_0 lx_1 - \alpha x_2 - r_M x_3 - \beta x_2^2 - \gamma x_2^3 + lB_1 x_1 x_2 + lB_2 x_1 x_2^2) \qquad (24)$$

Discretizing the above equation using the Euler approximation,

$$\left.\frac{dx}{dt}\right|_{t=kT} = (x(k+1) - x(k))/T$$

we have the following difference equation in state-space form

$$x(k+1) = \begin{bmatrix} a_{11} & 0 & a_{13} \\ 0 & 1 & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} x(k) + \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} u(k)$$

$$+ \begin{bmatrix} p_{11}x_2 x_3 + p_{12}x_2^2 x_3 \\ 0 \\ p_{31}x_2^2(k) + p_{32}x_2^3(k) + p_{33}x_1(k)x_2(k) + p_{34}x_1(k)x_2^2(k) \end{bmatrix}$$

$$y(k) = (0 \ 1 \ 0)^T x(k) \qquad (25)$$

where the terms indicated by zero or unity are always zero or unity, $u = e$, $a_{11} = 1 - Tr/L$, $a_{13} = -TlB_0/L$, $a_{23} = T$, $a_{31} = TB_0 l/m$, $a_{32} = -T\alpha/m$, $a_{33} = 1 - Tr_M/m$, $b_1 = T/L$, $p_{11} = -TB_1 l/L$, $p_{12} = -TB_2 l/L$, $p_{31} = -T\beta/m$, $p_{32} = -T\gamma/m$, $p_{33} = TB_1 l/m$, $p_{34} = TB_2 l/m$, and $x(k) = (x_1(k) \ x_2(k) \ x_3(k))^T$.

IV. NUMERICAL RESULTS

This section presents the simulation results of the adaptive linearization method on the loudspeaker model in Equation (25). The general scheme is depicted in Fig.5, where the adaptive nonlinear pre-processor can be implemented using DSP's. The loudspeaker had the following parameters

$$x(k+1) = \begin{bmatrix} -0.1 & 0 & -0.2 \\ 0 & 1 & 1 \\ 0.6 & -0.5 & -0.15 \end{bmatrix} x(k) + \begin{bmatrix} 0.4 \\ 0 \\ 0 \end{bmatrix} u(k)$$

$$+ \begin{bmatrix} -0.04x_2 x_3 - 0.05x_2^2 x_3 \\ 0 \\ -0.08x_2^3(k) + 0.01x_1(k)x_2(k) + 0.02x_1(k)x_2^2(k) \end{bmatrix}$$

$$y(k) = (0 \ 1 \ 0)^T x(k)$$

where the sample period T was set to be unity and the parameter β was chosen as zero, as in [12], since β is very small in practice. A reference linear filter having the linear parts of the loudspeaker model was used. The output of the reference linear filter was equal to the linear part of the loudspeaker output signal. Before the linearization started, the difference between the loudspeaker output and the reference linear filter measured the original distortion and after the linearization started, it measured the residual distortion. The input signal to the reference linear filter was delayed by δ samples due to the same amount of delay involved in the loudspeaker output before and after the linearization. The mean square values of the linear part and the nonlinear part of the loudspeaker output were $-10.0dB$ and $-39.4dB$, respectively.

The orders of the forward-modeling adaptive nonlinear FIR filter were $n_1 = 17$, $n_2 = 10$, $n_3 = 10$, the step sizes were $\mu_1 = 0.01$, $\mu_2 = 0.0001$, and $\mu_3 = 0.0001$ for the nonlinear filter. The order of the reverse modeling linear filter was 6, the step size was $\mu = 0.07$, and the delay in the inverse modeling was $\delta = 3$ samples. The initial coefficients of the adaptive filters were set to zero. As discussed before, the adaptive filters can not provide the good estimates of the operators at the beginning of the adaptation process, it will be better-off if the linearization process starts when the adaptive filters have converged. The linearization process started at $80k$ iterations. After $80k$ iterations, the MSE for inverse modeling by the linear filter was $-34dB$, which could not be reduced further by a linear filter due to existence of nonlinearity in the system. MSE for forward-identification by the nonlinear filter was $-67.6dB$ after $80k$ iterations. After the linearization took effect at $80k$ iterations, the nonlinear distortion was reduced from the original value of $-39.4dB$ to $-66dB$, that is, 5% of the original distortion. In other words, the ratio of the linear signal to the nonlinear distortion was increased to $56dB$ from $29.4dB$, namely, nearly doubled.

V. CONCLUSIONS

Nonlinear distortions in loudspeakers sometimes severely degrade the quality of sound reproduction. These distortions include nonlinearity in the suspension system and inhomogeneity in the flux density. A recently presented adaptive linearization scheme has been applied to linearizing a loudspeaker. A loudspeaker model embodying these two major sources of distortions is developed for a basic direct radiator loudspeaker. Simulations on this loudspeaker model have shown the promise of the proposed adaptive linearization method. Although the two major sources of nonlinearity of a loudspeaker are discussed in the paper, the method is also expected to work on a loudspeaker with other sources of nonlinearity. Further work includes experimentation of the method with measurement data and real-time implementation of the method with DSP processors.

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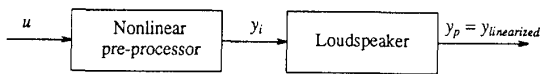[14] M. Rossi, *Acoustics and Electroacoustics,* Norwood, Massachusetts: Artech House, Inc., 1988.
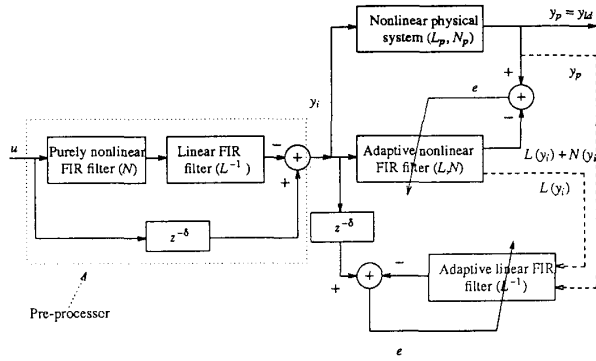
Fig.2 Adaptive implementation using FIR filters for the scheme with a pre-processor. Either one of the two dashed lines could be used. The linear FIR filter is copied from the adaptive linear FIR filter, and the nonlinear filter N is a copy of the adaptive nonlinear FIR filter.
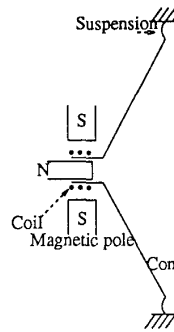

Fig.3 A conceptual structure of a basic radiator loudspeaker.

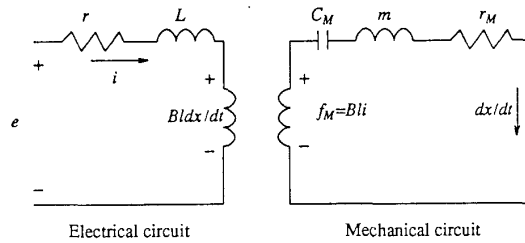
Electrical circuit   Mechanical circuit

Fig.4 Equivalent electrical and mechanical circuits of a loudspeaker.
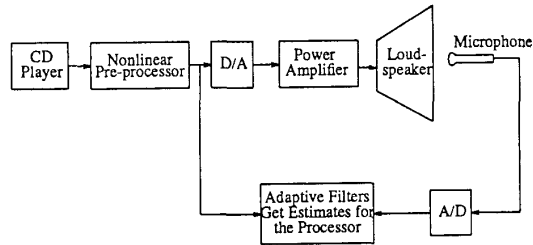

Fig.5 Adaptive Linearization of a loudspeaker using a nonlinear pre-processor.

Fig.1 A nonlinear pre-processor is placed at the input side of the nonlinear physical system to pre-distort the signal.