A multi-feedback design for LC bandpass Delta–Sigma modulators

Omid Shoaei and W. Martin Snelgrove
Department of Electronics, Carleton University
Ottawa, Ontario, Canada K1S 5B6
1-613-788-2381
omid@doe.carleton.ca & snelgar@doe.carleton.ca

ABSTRACT

A new technique for designing an LC bandpass Delta-Sigma modulator is presented. This method is based on pulse shaping of a DAC output signal such that one can realize a desired (arbitrary) loop transfer function. Especially for higher-order modulators where extra LC sections are added, sufficient number of parameters are provided in the feedback loop. It is shown that by creating more degrees of freedom one can achieve the maximum SNR in a given modulator order without constraining the noise transfer function of the modulator.

I. INTRODUCTION

Switched-C techniques have been used to implement bandpass Delta-Sigma analog/digital converters [1]-[3]. Continuous-time modulators can be far faster than their switched-C counterparts. The proper *s*-domain loop transfer functions for implementing a continuous-time equivalent from a given discrete-time (Switched-C) modulator have been recently reported [4]. A new architecture for a transconductor-C ΔΣ modulator has been given too [4]. For implementing a bandpass continuous-time ΔΣ modulator loop filter, however, the cascade of LC resonators as shown in Fig. 1 with

$$\frac{V_o(s)}{V_i(s)} = \frac{(gm/C)s}{s^2 + 1/LC} \qquad (1)$$

is attractive. Because 1) its architecture is simple, 2) a passive LC resonator has much less nonlinearity than an active resonator such as transconductor-C, and 3) LC type filters can present higher frequency capability than active filters. It is, however, difficult to construct linear high-Q LC resonators on-chip, so these converters have generally relied on off-chip inductors [5]-[7]. Since for a bandpass continuous-time ΔΣ modulator, a high-Q[1] resonator is required [4], for on-chip inductance implementation some Q enhancement technique [10] is necessary. The other problem is that the cascade of LC resonators shown in Fig. 1 provides a transfer function with a numerator having only bandpass term like the transfer functions implemented in [5]-[7]

$$\hat{H}(s) = \frac{ks^n}{(s^2 + \omega^2)^n} \qquad (2)$$

where n is the number of cascade stages, ω is the resonant frequency, and k is the overall filter gain. As shown in [4], in a continuous-modulator with order of $2n$ the proper *s*-domain loop transfer function numerator is a $2n-1$ th-order polynomial with non-zero coefficients having $2n-1$ distinct zeros, while (2) has n zeros at $s = 0$. For example, for a multiple-pole fourth-order system the loop filter [4] is

$$\hat{H}(s) = \frac{\left(\frac{\pi}{2} - \frac{1}{4}\right)\frac{s}{T}^3 + \left(\frac{3\pi^2}{16} + \frac{\pi}{4}\right)\frac{s}{T^2}^2 + \left(\frac{\pi^3}{8} + \frac{\pi^2}{16}\right)\frac{s}{T^3} + \frac{3}{4}\left(\frac{\pi}{2T}\right)^4}{\left(s^2 + \left(\frac{\pi}{2T}\right)^2\right)^2} \qquad (3)$$

In this paper we will address this problem in LC modulators. We will show how we can get the appropriate loop impulse response from a cascade of simple LC resonators by adding extra feedback loops.

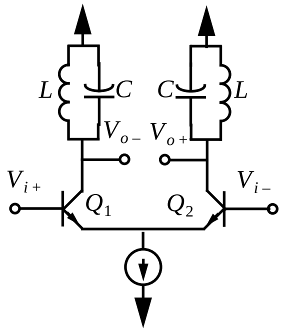

Fig. 1: A differential LC resonator.

II. MULTI-FEEDBACK DESIGN

We begin with a second-order bandpass case. The discrete-time loop transfer function ([2], [8]) is

$$\frac{z^{-2}}{1 + z^{-2}}. \qquad (4)$$

The loop impulse response of this system is a *cosine* waveform with its first two samples zero:

1. In [4] it is shown that in a fourth-order multiple-pole bandpass ΔΣ modulator for getting the maximum achievable SNR, the typical Q required is at least 50.

$$h(n) = \begin{cases} 0 & n = 0, 1 \\ \cos\left(\frac{(n-2)\pi}{2}\right) & n = 2, 3, \ldots \end{cases} \qquad (5)$$

In a continuous-time modulator the overall loop impulse response is obtained by convolution of the *s*-domain loop filter with the DAC impulse response. Three different possible DAC feedbacks are non-return to-zero (NZ), return to-zero (RZ), and half-delay return to-zero (HZ). Their impulse responses represented by $P_{NZ}(t)$, $P_{RZ}(t)$ and $P_{HZ}(t)$ are shown in Fig. 2. Hence, the overall loop func-

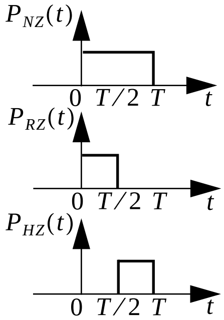

Fig. 2: NZ, RZ, and HZ DAC impulse responses.

tion in a continuous-time modulator employing a simple LC filter *i.e.* $\omega s/(s^2+\omega^2) = \boldsymbol{L}[h2(t)]$ where $h2(t) = \omega\cos(\omega t)$ and $\omega = \pi/(2T)$, for NZ, RZ, and HZ feedback pulses respectively are

$$\boldsymbol{Z}[P_{NZ}(t) * h2(t)|_{t=nT}] = \frac{z^{-1}(1-z^{-1})}{1+z^{-2}}$$

$$\boldsymbol{Z}[P_{RZ}(t) * h2(t)|_{t=nT}] = \frac{z^{-1}\left(\left(1-\frac{1}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}z^{-1}\right)}{1+z^{-2}} \qquad (6)$$

$$\boldsymbol{Z}[P_{HZ}(t) * h2(t)|_{t=nT}] = \frac{z^{-1}\left(\frac{1}{\sqrt{2}}-\left(1-\frac{1}{\sqrt{2}}\right)z^{-1}\right)}{1+z^{-2}}$$

As (6) shows the loop impulse response of a system including any feedback: NZ, RZ, or HZ by itself can not implement the required *cosine* loop impulse response given in (5). In particular, none of them provides a pure z^{-2} term to make the second sample zero. However, with a linear combination of any of two preceding feedback pulses given in (6), for example, RZ and HZ as shown in Fig. 3, it is possible to produce the desired second-order loop function $z^{-2}/(1+z^{-2})$. This requires finding two unknown coefficients from two simple linear equations. For example, for K_{rz} and K_{hz} from (6) and (4) the equality

$$K_{rz}z^{-1}((1-1/\sqrt{2})-(1/\sqrt{2})z^{-1}) + K_{hz}z^{-1}(1/\sqrt{2}-(1-1/\sqrt{2})z^{-1}) \equiv z^{-2} \qquad (7)$$

implies that $K_{rz} = -(1+1/\sqrt{2})$ and $K_{hz} = 1/\sqrt{2}$.

As shown in Fig. 3 there is no digital delay in the ΔΣ loop preceding the DACs. This represents a zero-delay continuous-time scheme [8], [9]. It should be noted that it is possible to have a second-order continuous-time system in which one delay is realized digitally [8], [9]. The coefficients of the zero-delay second-order system (shown in Fig. 3) for three different combinations of NZ, RZ and HZ pulses are given in Table 1. The corresponding coefficients for one-

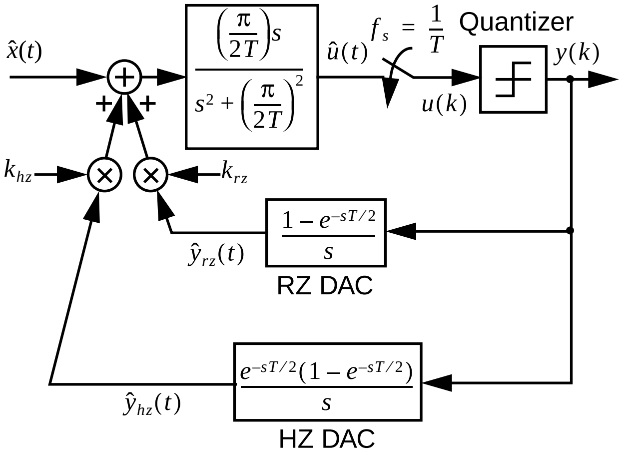

Fig. 3: A second-order multi-feedback (RZ and HZ) ΔΣ modulator with a LC resonator loop filter.

delay scheme is given in Table 1 too. Although the one-delay scheme will cost an extra flip-flop, the modulator sensitivity to extra non-ideal loop delays (propagation delay time in comparator and DAC,...) is reduced.

Table 1: Second-order LC modulator parameters.

Coefficients		Combinations		
		RZ–HZ	NZ–RZ	NZ–HZ
Zero-delay coefficients	k_{nz}		$1/\sqrt{2}$	$-(1+1/\sqrt{2})$
	k_{rz}	$-(1+1/\sqrt{2})$	$-1/(\sqrt{2}-1)$	
	k_{hz}	$1/\sqrt{2}$		$1/(\sqrt{2}-1)$
One-delay coefficients	k_{nz}		$1+1/\sqrt{2}$	$-(1/\sqrt{2})$
	k_{rz}	$-(1/\sqrt{2})$	$-1/(\sqrt{2}-1)$	
	k_{hz}	$1+1/\sqrt{2}$		$1/(\sqrt{2}-1)$

For implementing the fourth-order bandpass system from a cascade of two simple LC resonators, $\omega s/(s^2+\omega^2)$, as shown in Fig. 4 four coefficients are required. In Fig. 4 the shorter loops (the paths with $k2$ coefficients) each contains a resonator whose convolution with the corresponding feedback pulse results in the transfer functions given in (6). The longer loops (the paths with $k4$ coefficients) each includes a cascade of two resonators $\omega^2 s^2/(s^2+\omega^2)_2 = \boldsymbol{L}[h4(t)]$, where $h4(t) = 0.5\omega^2(t\cos\omega t + (\sin\omega t)/\omega)$ and $\omega = \pi/(2T)$. The transfer functions on these paths for NZ, RZ, and HZ feedback pulses respectively are

$$\mathbf{Z}\left[P_{NZ}(t) * h4(t)\big|_{t=nT}\right] = 0.25\frac{\pi z^{-1}(1 - z^{-1} - z^{-2} + z^{-3})}{(1 + z^{-2})^2}$$

$$\mathbf{Z}\left[P_{RZ}(t) * h4(t)\big|_{t=nT}\right]$$

$$= \frac{\pi z^{-1}(0.161612 - 0.265165z^{-1} + 0.015165z^{-2} + 0.088388z^{-3})}{(1 + z^{-2})^2} \quad (8)$$

$$\mathbf{Z}\left[P_{HZ}(t) * h4(t)\big|_{t=nT}\right]$$

$$= \frac{\pi z^{-1}(0.088388 + 0.015165z^{-1} - 0.265165z^{-2} + 0.161612z^{-3})}{(1 + z^{-2})^2}$$

The required multiple-pole loop transfer function for a

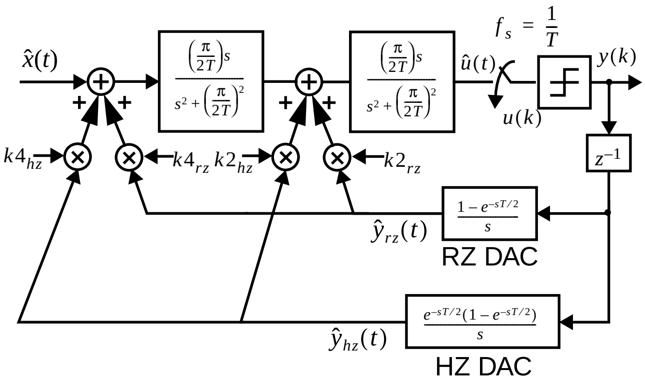

Fig. 4: A fourth-order multi-feedback (RZ and HZ) ΔΣ modulator with cascade of two LC resonator loop filter.

fourth-order system as shown in [3], [4] is $z^{-2}(2 + z^{-2})/(1 + z^{-2})^2$. The discrete-time loop impulse response of this system is

$$h(n) = \begin{cases} 0 & n = 0, 1 \\ -(0.5n + 1)\cos\left(\frac{n\pi}{2}\right) & n \ge 2 \end{cases} \quad (9)$$

which is shown in Fig. 5 with sample points represented by 'x'. It can be shown [9] that NZ pulse response of the multiple-pole fourth-order loop filter given in (3) results in the overall continuous-time loop impulse response, $h(t)$. As shown in Fig. 5 $h(t)$ is defined by $h_0(t)$ and $h_1(t)$ whereas

$$h(t) = \begin{cases} h_0(t) & 0 < t \le T \\ h_1(t) & T \le t \end{cases} \quad (10)$$

and for normalized $T = 1$

$$\begin{cases} h_0(t) = 0.75 - 0.25(3 + t)\cos\left(\frac{n\pi}{2}\right) + 0.25(4 + t)\sin\left(\frac{n\pi}{2}\right) \\ h_1(t) = (1.5 + 0.5t)\sin\left(\frac{n\pi}{2}\right) \end{cases}$$

It should be noted that because of a z^{-1} discrete delay factor inside the loop (Fig. 4) the overall continuous-time loop impulse response is shifted by T in Fig. 5. The continuous-

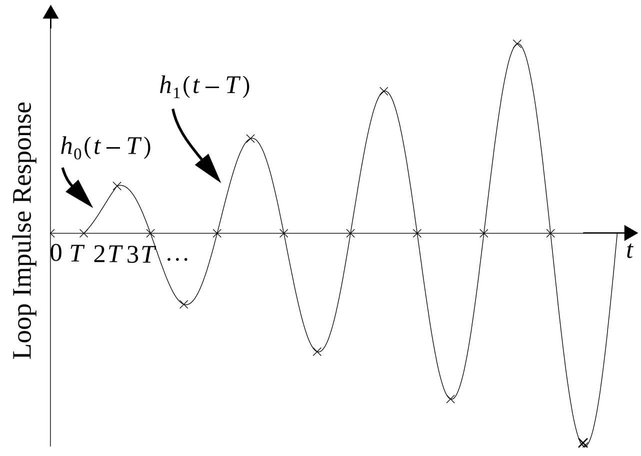

Fig. 5: Loop impulse response of the fourth-order multiple-pole modulator.

time loop impulse response (10) matches the discrete-time loop response (9) at the sampling times as shown in Fig. 5. The continuous-time loop filter shown in (3) can be implemented by a transconductor-C architecture directly [4]. For fourth-order LC modulator, however, it is obvious from (8) that none of the simple NZ, RZ or HZ modulators can implement this transfer function directly. However, from (6) and (8), it can be shown that with any combination of two pulses like RZ and HZ it is possible to build the ideal fourth-order loop transfer function. This requires solving four linear equations (two from (8) and two from (6)) to obtain the four unknown coefficients. The coefficients of the fourth-order multiple-pole system (with one digital delay inside the loop like the one shown in Fig. 4) for three different combinations of NZ, RZ and HZ pulses are given in Table 2. For instance, the four RZ and HZ coefficients shown in Fig. 4 are $k4_{rz} = -0.450158$, $k4_{hz} = 1.08678$, $k2_{rz} = -0.633883$ and $k2_{hz} = 2.98744$. It should be mentioned that there is a zero-delay solution [8], [9] for a multiple-pole fourth-order continuous-time modulator too. However, for practical reasons it is very sensitive to extra loop delays which reduces its usefulness especially for high speed applications, so it is not shown here.

III. SIMULATION RESULTS

The preceding second-order and fourth-order discrete-time and their corresponding LC systems are simulated to obtain the SNR. Although as mentioned, the loop transfer function of the given LC systems and their discrete-time counterparts are equal, their input signal transfer functions are different [8], [9]: $\pi/2$ in the second-order and $(\pi/2)^2$ in the fourth-order systems shown in Fig. 3 and Fig. 4 respectively. This signal gain difference causes the same SNR in

LC systems to happen at different input signal levels.

The maximum SNRs in a 2MHz bandwidth, with a sinusoidal input at 50MHz, for the second-order discrete-time and LC systems were 47.9dB and 46.4dB which occurred at input amplitude 0.49 and 0.39 respectively. For the fourth-order discrete-time and LC systems, the maximum SNRs in the same bandwidth and frequency were 65.4dB and 64.27dB which happened at input amplitude 0.49 and 0.31 respectively. The bit stream spectrum of the multiple-pole fourth-order LC modulator for a 0.31 input sine wave at 50MHz is shown in Fig. 6 (clock rate is at 200MHz).

Table 2: Multiple-pole fourth-order LC modulator parameters.

Coefficients		Combinations		
		RZ—HZ	NZ—RZ	NZ—HZ
Fourth-Order coefficients	$k4_{nz}$		1.08678	–0.450158
	$k4_{rz}$	–0.450158	–1.53694	
	$k4_{hz}$	1.08678		1.53694
Second-Order coefficients	$k2_{nz}$		2.98744	–0.633883
	$k2_{rz}$	–0.633883	–3.62132	
	$k2_{hz}$	2.98744		3.62132

IV. CONCLUSION

The design of a continuous-time LC bandpass ΔΣ modulator has been discussed. It has been shown that by employing a DAC pulse shaping technique it is possible to force the time domain response of a cascaded LC ΔΣ modulator loop to match that of the discrete-time ΔΣ modulator loop equivalent. The general architecture for a LC ΔΣ modulator with DAC pulse shaping is given. Adding two degrees of freedom at the input of each simple bandpass LC resonator section by means of pulse shaping allows complete control of noise shaping for an arbitrary ΔΣ modulator order. At any LC bandpass ΔΣ modulator with order of *2n*, the new *2n* unknown coefficients can easily be found by solving a set of *2n* linear equations. A second-order and a multiple-pole fourth-order modulator, the two most common examples, have been shown. The simulation results for these examples verified the theory.

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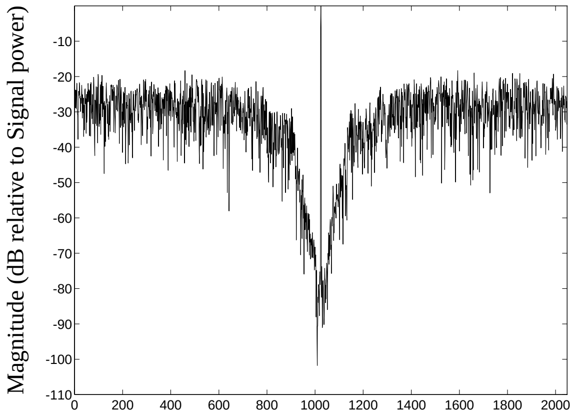

Fig. 6: The bit stream spectrum of simulated 4th-order LC modulator (input frequency is at 50MHz).

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