# An Efficient Adaptive Cascade IIR Filter

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**Abstract**

Among the various linear infinite impulse response (IIR) structures for adaptive filters, cascade IIR filters have advantages of easy stability monitoring and good sensitivity performance. However, their high cost for computing gradients is a problem. A novel technique of backpropagating the desired signal is proposed and then a cascade structure satisfying the requirements of this technique is developed, resulting in an efficient adaptive filter. It has been shown that equation-error formulation is just a special case of this backpropagation idea.

## 1 Introduction

An adaptive linear lIR filter has advantages in computation when a system is better modeled by a pole-zero transfer function than by a zero-only function, especially when poles are close to the unit circle in the z-domain. Several structures have been proposed for adaptive linear IlR filters, including direct form [l-41, lattice form [5-71, cascade-form [8-111, parallel-form [12,13], and recently, state-space structures [14]. Among them, the direct form is most popular in the literature. However, an adaptive filter may go unstable during adaptation and it is difficult to ensure stability of a direct-form filter with an order above two. It also has very poor sensitivity performance, which means that a slight change in a coefficient will result in a large change in filter output. This is undesirable for an adaptive filter since its coefficients are constantly affected by measurement noise and quantization noise. Both cascade form and parallel form have an easy stability check and low sensitivities. The parallel form has difficulties implementing a multiple pole.

A cascade IIR structure was developed for both the output-error formulation [10] and the equation-error formulation [11]. It implements the filter denominator in cascade form and the numerator in transversal form. An adaptive cascade filter, composed of IIR notch biquads, was developed for the output-error formulation in [8], which is suitable for detecting and enhancing multiple sinusoids in applications in communications and radar. Another cascade IlR filter was presented in [9] using the equation-error formulation, where the second-order sections are expressed in terms of their roots and these roots, rather than the section coefficients, are adapted.

One problem of adaptive cascade filters is the complexity of computing filter gradients, which is normally quadratic in the filter order. To solve this problem, an efficient cascade IIR filter is proposed based on a novel concept of backpropagating the desired signal. The filter consists of a transversal section and cascaded all-pole second-order sections. The computation for adaptation is about the same as that required by the filter itself when the LMS algorithm is used. It has been shown that the equation-error formulation is only a special case of the method of backpropagating the desired signal. The adaptive filter presented here has a similar structure to those in [ 10,111, but requires much less computation.

## 2 Backpropagation Formulation

### The Formulation

**The** popular output-error formulation minimizes the error computed at the filter output side. This section proposes a different scheme in which a desired signal is backpropagated and intermediate errors are generated, then the filter adjusts its coefficients to minimize the intermediate errors.

The complexity of gradient computation of a conventional output-error cascade filter is due to the fact that the filter objective is minimization of the error at the filter output and gradient signals of a section have to pass the subsequent sections to form gradient signals at the filter output side. If some kind of intermediate errors can be generated and intermediate errors, instead of the output error, are minimized, the computation will be more efficient. A structure with cascaded sections is shown in Fig.1. The transfer function Ti(z) can be arbitrary as long as its inverse is stable. The desired signal can be backpropagated into the system with cascaded inverse filter sections. Then, we can employ the intermediate desired signals to generate the intermediate error signals and adapt the coefficients. We now apply this idea to a cascade IIR filter.

If the desired signal is not backpropagated through the transversal section, the filter structure shown in the upper part of Fig.2 satisfies the stability requirement of the backpropagation method. The *nth* order filter is described by a transversal section

$$Y_{fir}(z) = H(z)U(z) \tag{1}$$

and $m$ all-zero second-order sections

$$Y_i(z) = \frac{1}{C_i(z)} Y_{i-1}(z) \qquad i = 1, 2, \cdots, m \tag{2}$$

where

$$H(z) = \sum_{i=0}^{n} h_i z^{-i},$$

$$C_i(z) = 1 - a_{i1} z^{-1} - a_{i2} z^{-2},$$

$$Y_0(z) = Y_{fir}(z),$$

and

$$Y_m(z) = Y(z)$$

The parameter $m$ is equal to $n/2$ if the order n is even, otherwise it is equal to $(n+1)/2$ and one of the "second-order" section is in fact a first-order section. The intermediate desired signals and the intermediate errors are generated as shown in Fig.2. The transversal section $H$ and the all-zero second-order sections $C_i$ can be adapted to minimize the intermediate errors.

It is clear that the coefficient vector $\mathbf{h}$ of the transversal section $H$ *can* be updated like that of an LMS transversal filter:

$$\mathbf{h}^{k+1} = \mathbf{h}^k + 2\mu_h e_1(k)\mathbf{u}(k) \tag{3}$$

where $\mathbf{u}(k) = (u(k)\, u(k-1) \ \cdots \ u(k-n))^T$ and $\mathbf{h} = (\, h_0 \ h_1 \cdots h_n)^T$.

Since the signals $D_{i+1}(z), D_{i+2}(z), \ldots, D_{m+1}(z)$ ( where $D_{m+1}(z) = D(z)$ ) are independent of the coefficients of the filter section $C_i$, the derivatives of the signal $D_i$ with respect to the coefficients of the section $C_i$ are nonrecursive:

$$\frac{\partial D_i(z)}{\partial \mathbf{a}_i} = -(\, z^{-1} \ \ z^{-2}\,)^T D_{i+1}(z) \tag{4}$$

where the vector $\mathbf{a}_i = (\, a_{i1}\, a_{i2}\,)^T$. These coefficients can also be updated like those of an LMS transversal titer:

$$\mathbf{a}_i^{k+1} = \mathbf{a}_i^k + 2\mu_{a_i} e_i(k)\mathbf{d}_{i+1}(k) \tag{5}$$

where $\mathbf{d}_{i+1}(k) = (\, d_{i+1}(k-1)\, d_{i+1}(k-2)\,)^T$.

Each all-zero second-order section $C_i$ is guaranteed to be stable and has a global minimum, although there is a possible bias in coefficient estimates when a measurement noise is present. An all-pole second-order section $1/C_i$ copies coefficients from its corresponding all-zero second-order section $C_i$, as indicated by the dashed lines in Fig.2. In the rest of the paper, the sections $C_i$ will be referred to as the all-zero second-order sections and the section $H$ will be referred to as the transversal section. The sections $C_i$

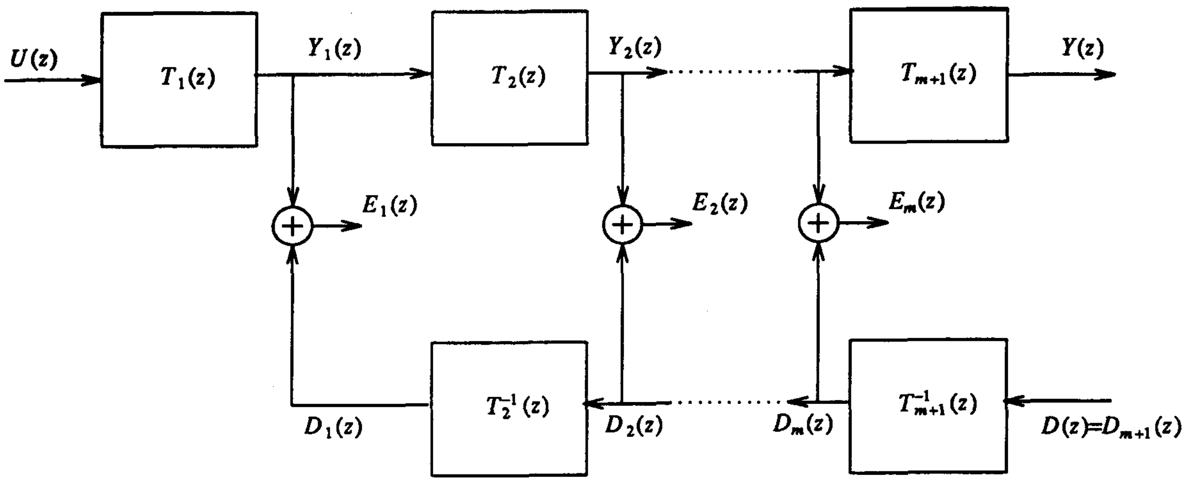


Fig.1 Backpropagation of the desired signal for a general cascade filter.

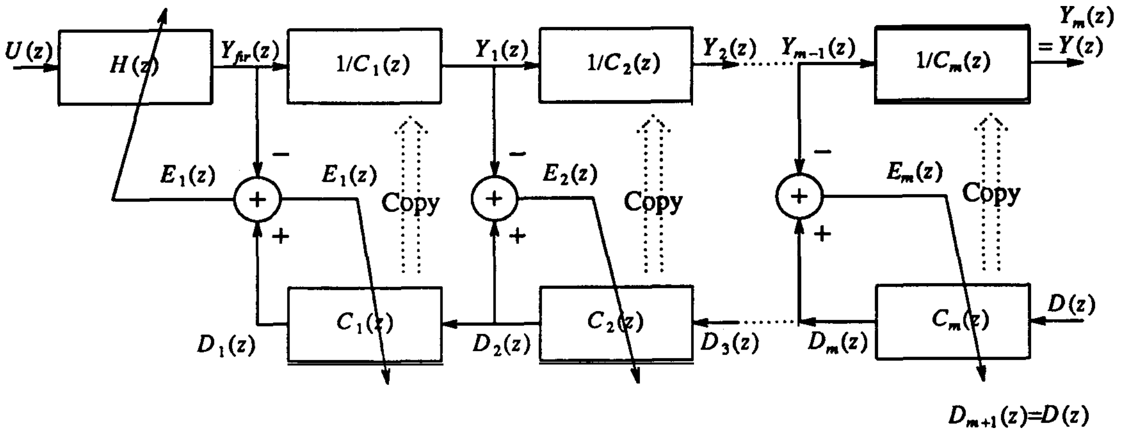


Fig.2 Adaptive Backpropagation Cascade Filter(BCF).

and $H$ *will* be collectively called the FIR sections.

### Stability Monitoring

**An** adaptive filter will be unstable if a pole of an adaptive IIR filter stays outside the unit circle long enough. This instability can be prevented by checking the pole locations. **One** major advantage of the cascade structure is its easy stability check.

It can be shown that the stability region of an all-pole second-order section is a triangle which is defined by [2]

$$1 + a_{i1} - a_{i2} > 0\,,\ \ 1 - a_{i1} - a_{i2} > 0, \text{ and } 1 + a_{i2} > 0. \tag{6}$$

This stability condition is monitored during adaptation. An unstable update might be corrected by reducing the step sizes. Once an unstable all-pole second-order section is detected in an iteration, the filter coefficients are computed again using smaller step sizes for the feedforward and feedback coefficients with the same gradients and error signal(s). If there is still at least one unstable all-pole second-order section, the filter coefficients will not be updated for that iteration. This is one of many possible ways of implementing stability monitoring and it is the one used in the simulations of this paper.

### Comparison with Equation-Error Formulation

**The** equation-error formulation was developed for a direct-form adaptive filter [2]. In the equation-error approach, the feedback signal is replaced by the desired signal so that the feedback coefficients are updated in an all-zero, nonrecursive form. The filter output is

$$Y(z) = A(z)D(z) + H(z)U(z) \tag{7}$$

where

$$A(z) = \sum_{i=1}^{n} a_i z^{-i}, \quad H(z) = \sum_{i=0}^{n} h_i z^{-i}$$

Fig.3 gives a popular pictorial description of the Equation-error Direct-form Filter **(EDF),** where $Y_{output}$ is the filter output. The error signal is

$$E(z) = (1 - A(z))D(z) - H(z)U(z) \tag{8}$$

which suggests Fig.4. Comparing Fig.4 with Fig.1, we find that Fig.4 shows that the equation-error formulation is just a special case of the backpropagation formulation illustrated in Fig. 1 when there are only two cascaded sections.

## 3 Simulation Results

**The** algorithms proposed in this paper have been simulated on a model matching problem, in which an adaptive filter attempts to match the transfer function of a reference system. A third order system has been used as a reference system in the simulations:

$$y(k) = 0.5765y(k-1) - 0.7810y(k-2) + 0.3821y(k-3) +$$

$$u(k) + 1.6751u(k-1) + 1.6751u(k-2) + u(k-3) \tag{9}$$

where the system poles are 0.5122 and $0.0321 \pm 0.8643i$ and zeroes are -1 and $-0.3375 \pm 0.9413i$.

For the cascade filter, the section $C_2$ is an all-pole second-order section whose optimal coefficient vector is $\mathbf{a}_2 = (\ 0.06429\ \ -0.748\ )^T$. **The** section $C_1$ is an all-pole first-order section whose optimal coefficient is $a_{11} = 0.5122$. The optimal coefficient vector of the transversal section $H$ is $\mathbf{h} = (\ 1\ \ 1.6751\ \ 1.6751\ \ 1\ )^T$.

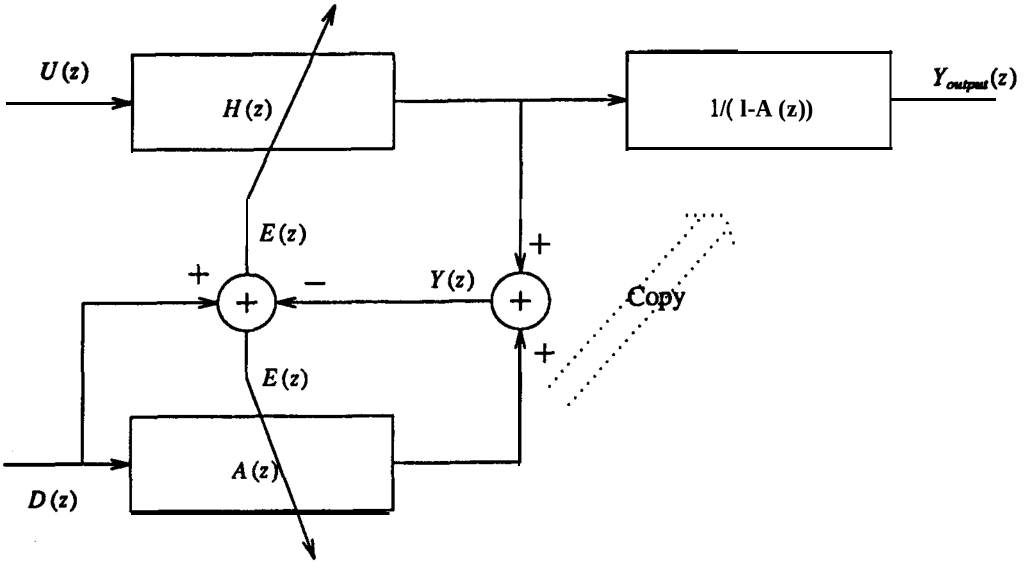


Fig.3 **Equation-error** fomulation.

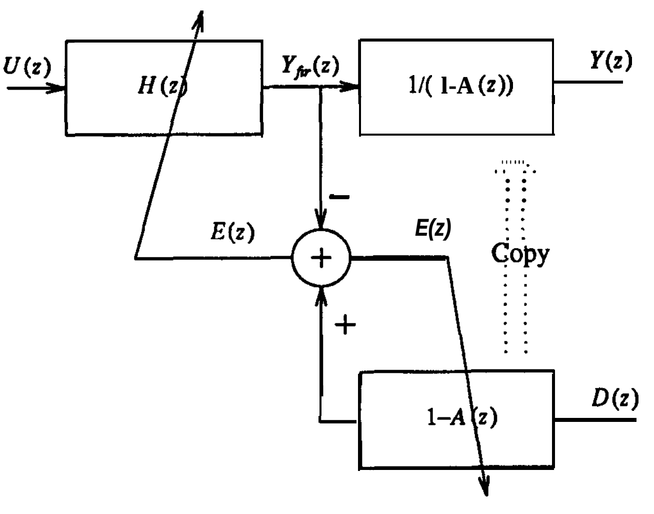


Fig.4 Alternative view of the **equation-error** formulation.

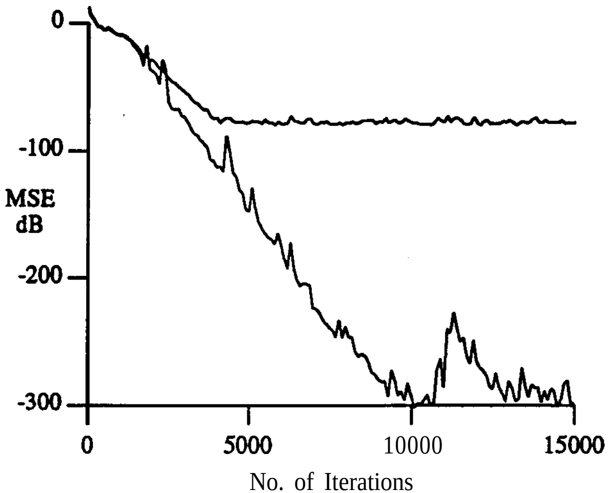


Fig.5 Convergence curves for the Output-error Direct-form Filter(ODF). Upper curve: additive noise of -80 dB, step size for IIR section = 0.0015 and step size for transversal section = 0.015. Lower curve: no additive noise, step size for IIR section = 0.002 and step size for transversal section = 0.03.

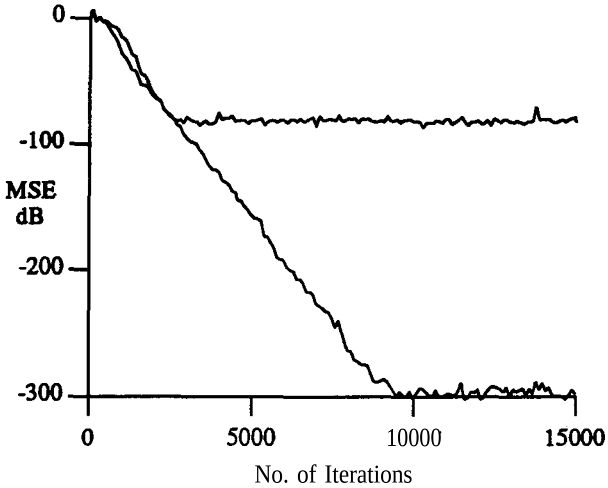


Fig.6 Convergence curves for the Backpropagation Cascade Filter (BCF). Upper curve: additive noise of -80dB, step size for all-pole second-order section = 0.004 and step size for transversal section = 0.049. Lower curve: no additive noise, step size for all-pole second-order section = 0.006 and step size for transeversal section = 0.09.

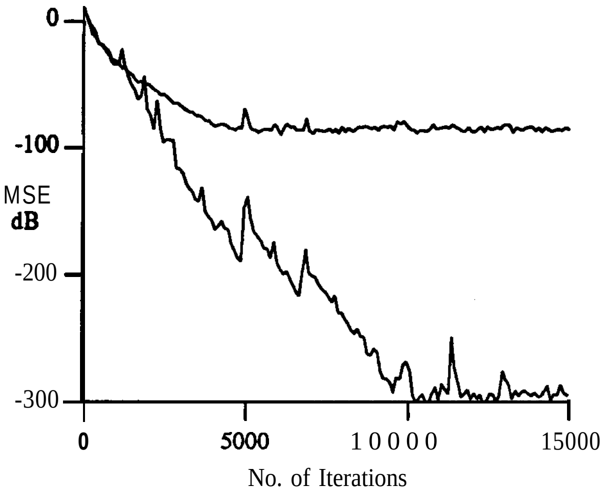


Fig.7 Convergence curves for the Equation-error Direct-form Filter(EDF). Upper curve: additive noise of -80 dB, step size for feedback section= 0.003 and step size for transversal section= 0.015. Lower curve: no additive noise, step size for feedback sectioo= 0.005 and step size for transversal section= 0.07.

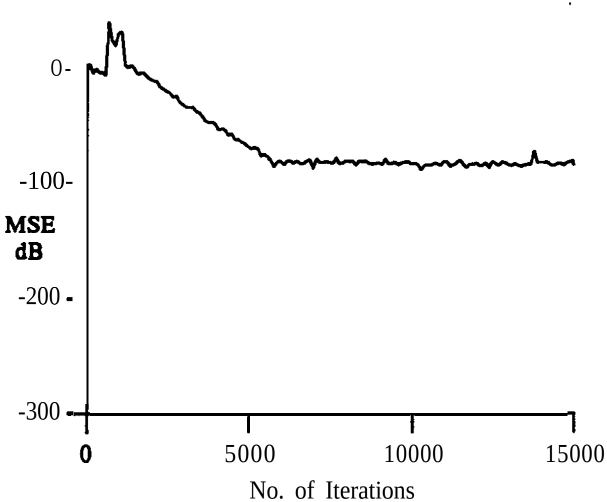


Fig.8 Convergence curve for BCF with measurement noise of 2dB. Step size for all-pole second-order sections = 0.004 and step size for transversal section = 0.049.

In all the tests, the mean square errors were computed using a data block of 100 samples. The input was a white Gaussian signal with unit variance ( $0dB$ ). The initial values of the adaptive filter coefficients were set to zero. Three sets of simulations have been performed using the three adaptive filters: Output-error Direct-form Filter (ODF), BCF of Fig.2, and EDF of Equation (7) or Fig.3.

In the first set of simulations, there was no additive noise on the reference signal (desired signal) and the step sizes were chosen so that the adaptive filters reached the computational noise floor (about $-300dB$) in the least number of iterations. The step sizes for the sections $C_1$ and $C_2$ were chosen the same for convenience. The convergence curves of the first set of simulations are the lower ones in Figs.57. Both the BCF and the EDF employ the backpropagated desired signals. So, it is interesting to compare the BCF with the EDF. Figs.6 and 7 show that the BCF had smoother curve and bigger step sizes. The ODF and the EDF had to use smaller step sizes because of the higher sensitivities of the direct-form structure. That the ODF and the EDF had spikier curves is also directly due to the higher sensitivities. These spikes are undesirable and although they can be reduced by using smaller step sizes, this will result in even slower convergence.

In practice, the reference signal is often contaminated by an additive noise, called measurement noise. An independent white noise of $-80dB$ was added to the reference signal to investigate the performance of the filters in the presence of measurement noise. The second set of simulations were performed under this condition. Suppose the adaptive filters are used to suppress echo in a data transmission channel. In such an application, the MSE is required to be less than about $-60dB$. Here, we require the MSE of an adaptive filter be below $-70dB$, allowing a safe margin. The step sizes were chosen so that the filters satisfied this MSE requirement in the least number of iterations. The convergence curves of the second set of simulations are upper ones in Figs.57. The BCF converged after $2.3k$ iterations. The EDF converged at $5.0k$ iterations, while the ODF at $3.8k$ iterations.

In the above simulations, no stability check was employed. Instability of an adaptive filter can occur, which might be caused by, for example, a surge of measurement noise, large step size, and/or large gradients due to steep performance surface. A third set of simulations were performed based on the second set of simulations. All the conditions in the third set of simulations were the same as those of the second set, except that there was a measurement noise surge from sample 600 to 1000. The ODF, the BCF, and the EDF went unstable without stability monitoring' when the measurement noise surge floor became high. Then stability monitoring was activated for the BCF, and the simulation was performed again. It remained stable and converged well. As expected, it worked well even if the noise level was very high. Fig.8 shows the convergence curve of the BCF with a noise of 26dB (standard deviation of 20), which shows a typical behavior of the BCF with stability monitoring. The filter worked normally before and after the noise surge. It had a high MSE level (but remained stable) during the surge because the gradient estimate was greatly corrupted.

## 4 Summary

This paper has studied adaptive cascade IIR filters which have an easy stability check and low parameter sensitivities. A novel concept has been proposed, which suggests backpropagating the desired signal through the inverse all-pole second-order sections and producing intermediate errors to be minimized. This concept was applied to a cascade IIR structure, resulting in an efficient adaptive cascade IIR filter. It has been shown that the equation-error formulation is just a special case of backpropagation of the desired signal. The convergence of the proposed filter has been analyzed and the results are presented in [ 151.

# References


[1] H. Fan and W.K. Jenkins, "An Investigation of an Adaptive IIR Echo Canceler: Advantages and Problems," ***IEEE Trans. on Acoustics, Speech, and Signal Processing,*** vol.36, pp.1819-1834, Dec. 1988.

[2] J.J. shynk, "Adaptive IIR Filtering," ***IEEE ASSP Magazine, pp.4 - 21,*** April 1989.

[3] C.R. Johnson, Jr., "Adaptive IIR Filtering: Current Results and Open Issues," ***IEEE Trans. on Information Theory, vol.IT-30,*** pp.237-250, March 1984.

[4] F.F. Yassa, "Optimality in the Choice of the Convergence Factor for Gradient-Based Adaptive Algorithms," ***IEEE Trans. Acoustics, Speech, and Signal Processing,*** vol. ASSP-35, pp. 48-59, Jan. 1987.

151 D. Parikh, N. Ahmed, and S.D. Stearns, "An Adaptive Lattice Algorithm for Recursive Filters," ***IEEE Trans. Acoustics, Speech, and Signal Processing,*** vol. ASSP-28, pp. 110- 112, Feb. 1980.

[6] N.I. Cho, C.H. Choi, and S.U. Lee, "Adaptive Line Enhancement by Using an IIR lattice Notch Filter," ***IEEE Trans. on Acoustics, Speech, and Signal Processing,*** vol. ***37,*** pp. 585-589, April 1989.

171 I.L. Ayala, "On a New Adaptive Lattice Algorithm for Recursive Filters," ***IEEE Trans. Acoustics, Speech, and Signal Processing,*** vol. ASSP-30, pp. 3 16-319, April 1982.

[8] T. Kwan and K.W. Martin, "Adaptive Detection and Enhancement of Multiple Sinusoids Using a Cascade IIR Filter," ***IEEE Trans. on Circuits and Systems,*** vol. ***36, pp.937-947,*** July 1989.

[9] Y.H. Tam, P.C. Ching, and Y.T. Chan, "Adaptive Recursive Filters in Cascade Form," ***IEE Proc.,*** vol. 134, Pt. F, Comm., Radar & Signal Processing, pp.245-252, June 1987.

[10] R.A. David, "A Modified Cascade Structure for IIR Adaptive Algorithms" ***Proc. of 15th Asilomar Conference on Circuits, Systems, and Computers,*** pp.175-179, Nov. 1981.

[11] R.A. David, "A Cascade Structure for Equation Error Minimization," ***Proc. of 16th Asilomar Conference on Circuits, Systems, and Computers, pp.182-186, Nov. 1982.***

[12] M. Nayeri and W.K. Jenkins, "Alternate Realizations to Adaptive IIR Filters and Properties of Their Performance Surfaces" ***IEEE Trans. on Circuits and Systems,*** vol. CAS-36, pp. 485-496, April 1989.

[13] J.J. Shynk, "Adaptive IIR Filtering Using Parallel-Form Realizations," ***IEEE Trans. on Acoustics, Speech, and Signal Processing,*** vol. ***37,*** pp. 519-533, April 1989.

[14] D.A. Johns, W.M. Snelgrove, and A.S. Sedra, "Adaptive Recursive State-Space Filters Using a Gradient Based Algorithm," ***IEEE Trans. on Circuits and Systems,*** vol. ***37, pp.673-684,*** June 1990.

[ 151 F.X.Y. Gao, ***Adaptive Filtering - Cascade IZR Filters, Linearization Techniques, and Nonlinear Recursive State-Space Filters,*** Ph.D thesis, University of Toronto, 1991.